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ABSTRACT

In this report, the author traces the recent development of standards for mathematics teaching. Two central tensions are examined: (1) the tension between arriving at consensus and effecting change; and (2) the tension between providing direction and affording discretion in practice. With a focus on the challenges inherent in the ambitious vision of teaching articulated in the National Council of Teachers of Mathematics "Standards," the author examines a case from her own teaching. The case is based on a lesson in which the author's third-grade students were working on a problem involving fractions. The story recounts the interactions that occurred during the lesson and the reflections of the author in response to those interactions. The report concludes with discussion of issues inherent in the notion of "implementation" of these new standards. (Author/MDH)

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Issue Paper 92-2

Implementing the NCTM Standards: Hopes and Hurdles

Deborah Loewenberg Ball

NCRTL

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IMPLEMENTING THE NCTM STANDARDS: HOPES AND HURDLES

Deborah Loewenberg Ball

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Abstract

In this report, the author traces the recent development of standards for mathematics teaching. Two central tensions are examined—(a) the tension between arriving at consensus and effecting change and (b) the tension between providing direction and affording discretion in practice. With a focus on the challenges inherent in the ambitious vision of teaching articulated in the National Council of Teachers of Mathematics *Standards*, the author examines a case from her own teaching. The report concludes with a discussion of issues inherent in the notion of "implementation" of these new standards.

Implementing the NCTM Standards:
Hopes and Hurdles[1]

Deborah Loewenberg Ball[2]

With increasing pressure to improve American students' mathematical competence, mathematics educators are trying—again—to change the practices and outcomes of school mathematics. Disappointed by the 1960s efforts at reform (e.g., Sarason, 1971; for an exception, see Romberg, 1990), the community watched the sharp shift back to "basics" in the 1970s. Some suspected that this movement was, at least in part, a reactionary swing from the "new math" with its emphasis on abstract mathematical structures. Then, in 1980, the National Council of Teachers of Mathematics (NCTM) published the *Agenda for Action*, outlining general directions needed to improve mathematics teaching and learning in the 1980s (NCTM, 1980). Although it was widely disseminated, like most documents of its ilk, the *Agenda* ultimately came to rest on many educators' shelves. A more ambitious move seemed necessary (Crosswhite, 1990). The two "standards documents" produced by NCTM (1989, 1991) over the past four years represent an unusual step to influence the character and quality of mathematics education.[3] One document focuses on curriculum and evaluation, the other on teaching, professional development, and the support and evaluation of teaching. Motivated by a desire to change the way mathematics is taught and learned in school, these documents move the discourse boldly behind the proverbial classroom door and provide new directions in both content and approach.

With the publication of these two sets of standards, we now face questions about what they can accomplish. In this paper, I use the second document—the *Professional Standards for Teaching Mathematics* (NCTM, 1991)—to explore the hopes and hurdles embedded in issues about how "standards"—and their evolution, dissemination, and

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[1]This paper was prepared for a conference on Telecommunications as a Tool for Educational Reform: Implementing the NCTM Standards, December 2-3, 1991, Aspen Institute, Wye Conference Center, Queenstown, Maryland.

[2]Deborah Loewenberg Ball, associate professor of teacher education at Michigan State University, is a senior researcher with the National Center for Research on Teacher Learning. The author would like to acknowledge the contributions of a number of her colleagues at MSU, especially David Cohen, Magdalene Lampert, Penelope Peterson, Dirck Roosevelt, Kara Suzuka, and Suzanne Wilson. She is also grateful to the other members of the NCTM Commission for the Professional Standards for Teaching Mathematics for all she gained from her work with them, especially Glenda Lappan, Thomas Schroeder, Thomas Cooney, and Susan Friel.

[3]The very idea of writing "standards" was doubtless a product of the broader political climate and the concurrent widespread interest in national standards and national examinations (Darling-Hammond & Wise, 1985; Porter, 1989).

implementation—may play a role in the reform of mathematics education.[4] Because the *Standards* are new, and their influence just beginning to unfold, I do not focus here on successful examples of implementation. Instead, I examine some conceptual issues that bear on the very notion of implementation itself.

What Is a "Standard"?

Initially, from a commonsense point of view, the notion of "standards" appears uncontroversial. Having standards seems worthwhile—whether we are talking about standards of behavior, product quality, or measurement. Still, our relationship with the idea of "standards" is nevertheless a deeply ambivalent one. In a society that values individualism, standards ring of standardization, control. And nowhere has the desire to codify expectations and ensure results confronted the fear of sameness and control as it has in education. Still, the rhetoric of standards—criticizing them, setting them, meeting them, raising them—increasingly dominates educational discourse.

Because "standard*ization*" implies sameness, standards are frequently seen as calls for quality via uniformity. However, this is only one, narrow view of the notion of a standard. A standard can also be "a rallying place," a *stand taken,* or a set of principles about what is valued. If teaching, as many are fond of saying, is an art, then standards for teaching may be able to function like standards in other arts. They may articulate standards of taste and judgment which do not determine a specific product or performance but which can guide the process of constructing and assessing that product or performance.

In the case of NCTM, the standards documents represent all of these ideas. As a vision, informed by multiple perspectives—including research knowledge, moral commitments, political motives, and philosophical orientations—the standards are intended to direct, but not determine practice; to guide, but not prescribe teaching. But this conception of standards, however palatable and sensible it may seem, creates tensions, tensions that have shaped their initial crafting, their ongoing interpretation and evolution, their impact on practice.

A Fundamental Tension: The Competing Need for Both Consensus and Change

Twin needs propelled the development of NCTM's standards for school mathematics: the need to gain consensus and the need to promote change. On one hand, if these standards were to stand as the banners of the community, then they had to reflect shared

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[4]Although I focus here on the standards about teaching, issues of curriculum and assessment underlie the issues I address.

values and commitments. On the other hand, if change was desired, then these standards had to do more than reflect current practice. New ideas were needed, ideas that departed from extant assumptions and practices.

This was not easy. Wide differences exist among NCTM members about what counts as good teaching and about the kinds of changes needed in schools. Mathematics educators disagree about things as fundamental as how mathematics and the physical or "everyday" world relate. They disagree about curriculum—such as the relative importance of number topics versus other mathematical ideas related to space, shape, or chance. They disagree about pedagogy, about whether and how to use manipulatives or what makes a good mathematics problem. They disagree about how much attention to give to developing skills—and even about what counts as "skill." And they even disagree about what mathematics is, and what it means to know, do, or use mathematics. NCTM's challenge was to create something around which mathematics educators could rally as a community. And they would need to be able to persuade a wider public of their stand, a public whose views of mathematics were likely to be more procedure and skill-oriented.

Because the *Professional Standards for Teaching Mathematics* is a written document, writing was a key site for the interplay of consensus and change. By custom, NCTM task forces are selected carefully, with an eye to representing the professional, geographic, gender, and racial diversity of the membership. The writing groups include experienced teachers, researchers, and teacher educators, from a variety of settings, and with diverse kinds of professional experience. The working groups convened to develop the *Standards* had among their members some of the resources to interweave the tried-and-true with the novel, idealism with realism. Contributors brought different ideas, ways of talking and thinking, as well as different images and commitments.

There were countless arguments about words, the prominence of different ideas, and the style of the presentation. A first draft was hammered out of the diverse points of view represented in the group. A draft of the document was then duplicated and distributed to thousands of educators all over the United States and Canada. Thousands of responses poured in. Should the working groups have been pleased when they read agreement with the draft ideas? Or should they have been pleased with disagreement? If people agreed with everything, then the writers were doubtless failing to provoke sufficient dissonance for change. Some of the ideas embedded in the draft were controversial, and unlikely to elicit such agreement. Still, if people rejected the ideas, the writers risked failing to create a sense of common direction. The ideas contained in the draft had to inspire both new thinking and the concurrence necessary for significant change to occur. The document had

to engender commitment; no official authority would support the ideas contained within. These standards would have to gain their authority through the persuasiveness of the ideas (Porter, 1989).

Even with the document revised and published, to say that there is consensus is, to some extent, misleading. The *Standards* represent a banner, not a dogma. The same diversity that went into constructing these standards remains. Diverse interpretations and enactments of the vision are inevitable. How diverse these are will in turn, however, shape the coherence of the change effort. This is an inherent tension of enactment. Community does not imply unanimity, any more than standards imply uniformity. This raises the spectre of another tension, a tension of how to provide guidance tempered by respect for professional autonomy.

A Second Essential Tension: Direction Versus Discretion

On one hand, particular change requires rather clear direction and guidance. On the other hand, teaching is context-specific. Teachers are professionals who must make professional judgments based on expertise, insight, and skill. Research on teaching highlights the centrality of such judgment, exercised within specific contexts. Even though it might make policymakers unhappy, *no* so-called "effective" practices are unilaterally appropriate across particular cases. Good teachers must work within a repertoire of possibilities, making decisions in the context of competing concerns and demands (Darling-Hammond & Wise, 1985; Shulman, 1983). The context-specific nature of teaching practice creates a challenge for those who would work for significant change in schools (Richardson, 1990).

Shulman (1983) describes this challenge, positing that initiatives for change "must be designed as a shell within which the kernel of professional judgment and decision making can function comfortably" (p. 501). He argues that such initiatives *cannot* determine directly teachers' actions or decisions,and he concludes that they can, at best, "profess a prevailing view, orienting individuals and institutions toward collectively valued goals, without necessarily mandating specific sets of procedures to which teaches must be accountable" (p. 501). This view of policy and its role in shaping teaching and the discourse around teaching—was implicit in all of the work in developing the NCTM *Professional Standards for Teaching Mathematics*.

One typical interpretation of this position is that, in order to improve students' learning, we should strive for consensus about the "what" that students should learn—the scope and sequence of topics—but leave the pedagogy of the curriculum up to teachers

(Porter, 1989; see also Schwille, et al., 1983). Teachers, administrators, and policymakers alike have tended to agree on this distinction and on its consequent division of authority for educational decisions. But this agreement, however politically comfortable, rests on a serious conceptual fallacy. Attending little to the realities of teaching and learning, this position separates *content* (what is taught) from *method* (how it is taught). Joseph Schwab (1978), echoing Dewey, wrote that "methods are rarely if ever neutral. On the contrary," he wrote, "the means we use color and modify the ends we actually achieve through them. How we teach will determine what our students learn" (p. 242).

At first, the NCTM appeared to fall into this traditional separation of content and method. The first document, *Curriculum and Evaluation Standards* (NCTM, 1989) directly addressed two elements commonly thought to dominate what students learn in school: the curriculum (what is taught) and assessment (the way we judge what is taught) (Resnick & Resnick, 1985). By implication, however, much *was* said about teaching in this document—in part, *because* content and method are fundamentally intertwined. Reading the *Curriculum and Evaluation Standards*, one could envision actual classrooms.

In the words of one reader, the curriculum standards document implied

> a room fully equipped for mathematics instruction . . . with an arrangement and atmosphere conducive to activity learning . . . a teacher who guides, questions, discusses, clarifies, and listens more than they [sic] lecture or give directions . . . a group of students who explore, investigate, discuss, reason, validate, represent, and construct mathematics . . . a curriculum that is rich in problem-solving activities." (Crosswhite, 1990, p. 465)

Ernest Boyer (1990), not a mathematics educator himself, noted that the document supports "active inquiry, multiple ways of solving problems, the use of manipulatives, and cooperative learning." But, he argued, far more attention needed to be given to pedagogy. He wrote: "It is not enough to suggest active learning and cooperative practices without greater clarity about how teachers might move constructively in those directions." And he called for "a good description of practice that moves in the direction of the reforms" (pp. 563-564)—in a language and a form that would be well-understood.

The idea to produce a complementary document—the *Professional Standards for Teaching Mathematics* (NCTM, 1991)—grew from observations like Boyer's. The second document's aim was to provide more guidance to those involved, at various levels, in changing mathematics teaching—teachers, teacher educators, supervisors, administrators, policymakers. Acknowledging directly that "what students learn is fundamentally connected

with how they learn it," the text begins by departing from the traditional separation between authority for content and autonomy for method:

> Students' opportunities to learn mathematics are a function of the setting and the kinds of tasks and discourse in which they participate. What students learn—about particular concepts and procedures as well as about thinking mathematically—depends upon the ways in which they engage in mathematical activity in their classrooms. Their dispositions toward mathematics are also shaped by such experiences. Consequently, the goal of developing students' mathematical power requires careful attention to pedagogy as well as to curriculum. (p. 21)

The charge for this second document was to make more explicit what lay between the lines of the curriculum standards. For example: In what kinds of roles and reasoning might a mathematics teacher engage in order to make possible such learning? What were the features of the mathematical activities illustrated throughout—and what might teachers consider in designing or selecting student tasks? What kind of atmosphere was important? In an bold position, NCTM (1991) focused directly on practice, all the while acknowledging its inherent complexity:

> Good teaching demands that teachers reason about pedagogy in professionally defensible ways within the particular contexts of their own work. The standards for teaching mathematics are designed to help guide the processes of such reasoning, highlighting issues that are crucial in creating the kind of teaching practice that supports the learning goals of the *Curriculum and Evaluation Standards*. This section circumscribes themes and values but does not—indeed, it could not—prescribe "right" practice. (p. 22)

With this rhetoric, the NCTM had taken a bold step, committing itself to move inside the classroom door, into the usually discretionary spaces of teachers' practice. In so doing, the NCTM standards require that we rethink the ways in which we balance concerns for guidance and autonomy in teaching and its reform.

Problems of Realizing Ambitious Visions of Teaching

A major source of the challenge presented by the *Professional Standards for Teaching Mathematics* (NCTM, 1991) is its ambitious vision. It holds out the hope of a richer mathematical curriculum, a curriculum aimed at developing all students' abilities to reason, solve problems, and communicate mathematically. Paper-pencil computation and algorithms

would not dominate classrooms as they have. Instead, skill and accuracy would have a place in the context of framing and solving a variety of "pure" and "applied" mathematical problems and investigations. Alongside the traditional emphasis on arithmetic, the curricular terrain would expand to include investigations of space, data, and chance. But this reform agenda is not just about reaching new agreements about what should be *taught*: It is also about what students should *learn*.

The recognition that the *what* is fundamentally tied up with the *how*—that content and method are intertwined—heightens the challenge of the *Standards* vision. The quality of the tasks is crucial, but, equally important is the nature of the classroom discourse—the ways in which ideas are developed interactively in the class. And the environment of the class—the kinds of norms that are established, the ethos of collaboration and respect, the patterns and expectations for thinking and interacting—combines with this attention to discourse, considerably extending and complicating what counts as mathematics pedagogy. Emphasizing reasoning in the use and development of mathematical ideas means involving students in teaching and learning as they rarely have been before. Rather than the teacher being the source of all knowledge, the teacher's role becomes more one of structuring the context in ways that help students construct and work with important mathematical ideas.

But no tight implications for practice can be inferred. For example, the vision painted in the *Standards* does not imply that teachers never tell, never present, never explain. In fact, the standard on the teacher's role in discourse makes clear how complex are the decisions that confront teachers as they interactively negotiate the development of ideas with and among students:

> *Standard 2: Teacher's Role in Discourse*
>
> **The teacher of mathematics should orchestrate discourse by—**
>
> - **posing questions and tasks that elicit, engage, and challenge each student's thinking;**
> - **listening carefully to students' ideas;**
> - **asking students to clarify and justify their ideas orally and in writing;**
> - **deciding what to pursue in depth from among the ideas that students bring up during a discussion;**

- **deciding when and how to attach mathematical notation and language to students' ideas;**

- **deciding when to provide information, when to clarify an issue, when to model, when to lead, and when to let a student struggle with a difficulty;**

- **monitoring students' participation in discussions and deciding when and how to encourage each student to participate.** (NCTM, 1991, p. 35)

Few people disagree with these dimensions of the teacher's role. Who can disagree that teachers must decide "when to provide information, when to clarify an issue, when to model, when to lead, and when to let a student struggle with a difficulty"? Or that teachers must monitor students' participation in class and thoughtfully find ways to engage each child? It is not the ideas that are so controversial; it is working with them from day to day, in the context of the complexity of classroom teaching, that sets up the hurdles that we face.

Because I think this argument is at the heart of any consideration of "implementing the *Standards*," I will tell a brief story from my own teaching of third-grade mathematics as a means of illustrating the challenges posed by this vision of teaching.[5] For the purposes of this paper, I tell my story to make more vivid what working with these ideas means in daily classroom life.

Briefly, a bit of context: For the past several years, I have been teaching mathematics daily to a heterogeneous group of third graders at a local public elementary school.[6,7] Sylvia Rundquist, the teacher in whose classroom I work, teaches all the other

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[5]My point here is not to argue that NCTM's vision is unreasonable or undesirable. Such arguments can and will be made. Questions will be raised about who decides what are worthwhile aims for mathematics education, and there will be bristling at the way in which this vision, in its very conception, crosses inside classroom walls. These arguments grow directly from the tensions I discussed above—the tensions of consensus and change, of direction and discretion. Others will be skeptical that teachers will be able to do this. Because I want this paper to contribute to ongoing discussions of what it would take for teachers to be able to develop their practice in the spirit of these ideas, I concentrate here on this last concern—a concern that centers on support for teachers and teaching.

[6]Several of my other Michigan State University colleagues are involved in this kind of work in classrooms, among them: Magdalene Lampert, Suzanne Wilson, Kathleen Roth, Daniel Chazan, and David Wong. Several of the doctoral candidates have also developed lines of research and practice that involve ongoing classroom teaching, among them: Ruth Heaton, Margery Osborne, Janine Remillard, and Kara Suzuka.

[7]Many students are from other countries and speak limited English; the American students are diverse ethnically, racially, and socioeconomically and come from many parts of the United States. This lesson took place during 1989-90. In this particular class, in which we had 22 students, 10 were from the United States and 12 were from other countries—Indonesia, Taiwan, Korea, Nepal, Nigeria, Kenya, Egypt, Ethiopia, Nicaragua, and Canada. Four of the 10 U.S. students were African-American.

subjects besides mathematics.[8] I use this teaching as one means for developing insight into what it takes to teach in the spirit of the current reforms—in ways that are responsive to children and responsible to curricular goals. I am interested in the dilemmas that arise, and in alternatives for managing these.[9] I am interested also in contributing to conversations like the ones represented by this conference, conversations centered on the kinds of supports and opportunities that are needed in order to foster real changes.

In this particular lesson, on a warm mid-May afternoon, my third graders were working on the following problem:

> $\frac{3}{4}$ of the crayons in Mrs. Rundquist's box of a dozen crayons are broken.
> How many unbroken crayons are there?

We were midway into an investigation of fractions (see Ball, in press-a). At this juncture I was trying to help the students develop a sense for fractions as representations for part-whole relationships where the whole is not necessarily one individual thing; that is, in school, most of their encounters with fractions seem to center on shading parts of wholes, for example,

Shade ***one fourth*** of the pie.

I had noticed that many students thought that one-fourth was necessarily less than one. Some even thought one-fourth was a *shape*:

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[8]Rundquist and I meet regularly to discuss individual students, the group, what each of us is trying to do, the connections and contrasts between our practices. We also spend a considerable amount of time discussing and unpacking mathematical ideas, analyzing representations generated by the students or introduced by me, assessing the roles played by me and by the students in the class discussions, and examining the children's learning.

[9]Magdalene Lampert's work on dilemmas of teaching has been a central inspiration for my ongoing interest in the challenges of teaching elementary school (see Lampert, 1985; see also Ball, in press-b).

So I had created this small problem because I hoped that it would help us confront the importance of the *whole*. Here three-fourths would not mean three-fourths of *1* crayon but three-fourths of *12* crayons—*9* crayons. Just to get this far, I had to consider very carefully what I knew about my students, what I knew about children of this age and their understandings of fractions more generally. I also had to understand different interpretations of fractions myself. The issue here is less that I made up my own problem—the same insights would be required to choose and use skillfully a task from a textbook.

In this story, however, rather than concentrate on the thinking I had to do in order to *prepare* for this lesson, I want to focus on the *interactive demands* that this kind of teaching places on the teacher. The story is intended to highlight what is implied by the ideas represented in the standard on the teachers' role in discourse in moment-to-moment teaching.[10]

The problem was written on the chalkboard:[11]

> $\frac{3}{4}$ of the crayons in Mrs. Rundquist's box of a dozen crayons are broken.
> How many unbroken crayons are there?

We begin our discussion of the problem after the children have spent almost 20 minutes working on the problem, first alone, and then with a partner or in a small group.

A tall boy named Sean volunteers to show his solution. "It would be four," he asserts as he come up to the board. He draws 12 sticks to represent the 12 crayons, and marks off groups of 4 crayons:

He explains, "Well, I um counted these and I got, I went 1, 2, 3, 4 and I put a line down. So it's . . . then I went 1, 2, 3, 4 and I put another line down and I add them up and it's 8, and I put another line 1, 2, 3, 4. And that was 12," he finishes.

I always wonder about whom to call on to start off our discussion of a problem. Who should have the floor first and why?

Accepting drawings like this one seems important to expand the tools that students can use to think with as well as to express their thinking.

I was glad to see that Sean expected that part of showing his solution was to explain what he did and what he was thinking.

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[10]On the left-hand side, I narrate a segment of the class discussion; on the right, I have annotated the narrative in order to call attention to some of the issues that I found myself confronting. This format is the format the we used to portray the vignettes in the *Professional Teaching Standards*.

[11]This example also appears in Ball, 1991.

"Why—" I begin to ask, but Sean interrupts, changing his mind. "A quarter wouldn't be *that*." He erases the lines, "Because um, because that's a *third*. There's only three groups. There's supposed to be *four* groups. Sean draws lines to mark off four groups of 3 crayons.

He explains: "Because it's three-fourths, that's what I said, it's three-fourths so 3 crayons is a fourth, so 3 and that's a fourth, that's a fourth and that's a fourth, so that's three-fourths.

Riba, waving her hand, disagrees. She says that one-fourth should have 4 crayons in the group—like Sean had presented it at first. "This is what I think: three-fourths is like, um, *three groups of 4*."

I decide to ask for other students' reactions. Ofala says she agrees with Sean. "I think he's right because he's taking the 3, like separating the three groups plus the one group he didn't circle."

I probe: "Why does he have 3 in every group instead of 4 in every group?"

Sean says that if it was three groups of 4, "this should have 4 in each one, and it would be *16*." (In other words, if three groups of 4 was the answer to three-fourths, it would have to be three-fourths of 16, not 12.)

This is always interesting to me when students figure out for themselves that something doesn't make sense. How to set up the environment so that kids feel comfortable changing their mind is a big concern, since in school being "wrong" is traditionally something to hide or to be ashamed of.

Should I have said something here? Many teachers would praise him for his explanation and for figuring out and revising his answer. I would like, though, for the students to come to rely less on me for confirmation and more on themselves.

Should I have asked Riba to say first what she thinks of Sean's explanation, partly to help her learn what it means to show respect for his ideas, and partly to make sure she really understood what he was saying?

The teacher's role in orchestrating discussions is so hard! Should I have clarified what the "4" means in $\frac{3}{4}$ *or should I have let the other students say what they were thinking?*

Here I decided that Riba's idea that $\frac{3}{4}$ *implies groups of four was worth probing more. Did this make sense to do this here—to open up the discussion to a "wrong" idea when it seems to be moving in a "right" direction?*

He erases the extra four lines and turns to Riba. Pointing at the drawing, he says, "These aren't fourths, these are thirds because there's three groups and that makes them a third."

Keith raises his hand, and explains that what Riba is saying is that one-fourth means "one group of 4." Riba nods. Sean turns to the class:

> Let's take a vote! How many people, um, think that my answer is correct, raise their hand; and how many people think Riba's answer is correct, raise your hand.

This seemed to be becoming competitive—a situation in which Riba's ideas are pitted against Sean's and in which there will be a "winner." Voting has been such a big part of their experience in settling group matters. Yet figuring out what makes sense in this case did not seem to be a matter of democratic vote. Should I just have explained this to him? I decided to ask him what he is thinking.

T: Why would that be a good idea? What would that do if we saw that? Why would we want to know that?

Sean: That would prove it.

Students: No! No!

What was Sean's notion of what makes something right or true in mathematics?

T: Keith, you're shaking your head. Why wouldn't that prove it?

Keith: Just because like, just because somebody agrees with another person doesn't mean that they're *right*.

Betsy: I have an example of why voting doesn't work because when we were talking about zero, if it was an odd or even. A whole lot of people said that it was an odd but then afterwards we figured out that it was even and voting didn't help us know if it was odd or even because the answer was opposite than what people had voted.

This seemed to be a good example from their shared experience that may have helped the students understand why knowledge isn't simply legislated.

T: So how did we change our minds then if the voting doesn't work?

Betsy: Because the people found out patterns and the number line and they figured out that no, zero must not be a odd because when it goes up there it goes odd, even, odd, even, odd, even and so when you had an odd number like one and then you have zero, zero must be even because that's the way it is.

T: Anybody else want to comment on this before we go back to our problem of fourths and thirds? Mei?

Mei: I don't think it would work, but it would be fun to see how many people agree with him because maybe some people would come up with some other idea.

T: So, you'd be curious just to know what people are thinking?

With a sudden start I realized how often I have "polled" the class—not to settle matters of disagreement, but to give myself and the children some information about the distribution of ideas in the group. I do not think of this as "voting"—that is, as a means to determine the correct answer. But this distinction is a subtle one and I did not know what the students might have been thinking about the role of voting.

Mei: Yeah.

Sean: I agree. But that's a really hard question that Riba is asking, but why shouldn't there be four groups of, um, 3.

I suggest that we return to trying to interpret what three-fourths might mean. Betsy volunteers that she has an idea.

I felt like Betsy, who tended to take the floor a lot, was doing too much of the talking in this lesson. Should I have done something here to get the ideas of other kids, or should I have called on her to see what she would say and then go from there? It seems that on different days, different students are more active than others. I am never sure how much to press on this—I do understand that students can be very engaged without speaking. Still, a big issue is how to keep an accurate sense of who is tuned into what, and what the patterns are across days. I want to encourage different children to participate and to be as thoughtful as possible about providing varied opportunities for participation.

Betsy: I'm thinking about what's a fraction that you know is true? A fraction that you know, that we already agreed—just wait. . . . *(pauses, thinking of an example)* Okay, yesterday people agreed on half of 24 was 12, right? *(to Riba)* Do you agree with that? Half of 24 is 12? Well, if we put 24 lines, we don't circle *two* in each group, do we? We went 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12. Then we cut it right there and we circled this half and that would be half.

Betsy had connected the question what the "4" means in $\frac{3}{4}$ to what the "2" seems to represent in the more familiar $\frac{1}{2}$. It is noteworthy to me that she is trying to convince Riba and the other students based on something that she says they "already agreed" on.

See, we have, we have *two* groups here, right? This *(pointing to the 2 in $\frac{1}{2}$)* means the *groups*.

Should I have gotten more students to respond to this? Should I have polled the class and see if there was some consensus around this idea? Was there a way I could have helped the students understand the difference between taking a poll in a discussion and making a decision about an idea based on a vote? Should I have taken the opportunity to amplify what she was saying and underscore her point—thereby indicating that it is right (given a part-whole interpretation of fractions)?

Within the short space of these few minutes of classroom time, I faced a series of issues: how to get and maintain all my students' engagement, how to make sense of what Sean and Riba were thinking, how to help them move toward appropriate and connected understandings of fractions. From moment to moment I was having to consider whether to praise, explain, solicit others' ideas, let an issue grow, or even stir up trouble in order to press on a crucial mathematical point. My teaching was in many ways consistent with the kinds of ideas promoted in the *Standards*: The standard on the teacher's role urges me to listen, to ask students to clarify their ideas, to pursue questions that will extend their thinking, to decide what to follow up on. Still, placing the text of the standards alongside a few minutes of classroom work makes clear that while the *Standards* provide guidance for my work, they do not—indeed, *cannot*—prescribe it. Day after day in my classroom, students

say things I had never considered. Day after day they have trouble with ideas I used to think were simple. And day after day, these eight-year-olds catch me off guard with what captures their interest and what they reach for.

The visions of teaching articulated in the two NCTM standards documents are ambitious (see Cohen, 1989). Three basic observations underlie my concerns in any talk of "implementation." First, this kind of teaching itself is hard, and no one is going to produce a system, or a formula, or a program that can produce it. There are no recipes for helping students construct useful and worthwhile understandings of mathematics in ways that connect with and extend their everyday worlds. In Porter's (1989) terms, the best that we can expect is that the *Standards* create a "context of direction" for change. Second, teachers—with whom much of this reform agenda lies—are being asked to create opportunities for learning mathematics that they have likely never had. Constructing ways to teach mathematics that take these ideas seriously will require new learning, support, and resources of new kinds.

Third, even if imaginative supports for learning and development could be conceived and made available in effective ways, the broader context in which the *Standards* are nested may undermine the work. For instance, as demands for accountability grow, teachers' latitude to experiment, to try new things, may be hampered. It seems paradoxical: In some sense, teachers are being urged to make their work yet more uncertain, even as they are simultaneously being asked to produce, more reliably, a set of ambitious outcomes. We want students to reason, to solve complicated problems, to perform intellectually challenging work. And, at the same time, we are creating tests to assess and monitor teachers' attainment of such ambitious goals. And, in general, societal support for such goals is ambivalent: The public wants students to be able to reason but also expects "math" in school to include all the things they remember from their own schooling. Tradition pulls conservatively on the reform agenda, leaving teachers uncertain about the space they have to make the changes articulated in the *Standards*.

My conclusion is a simple one: If this reform movement is to have any promise, resources and supports of a variety of kinds will be absolutely crucial to working with, toward—and beyond—the ideas represented in the *Standards*. It is to those supports that I turn in the last section of this paper.

Supporting Change

The reforms ask teachers to teach a mathematics they never learned—and in the case of the teacher educator, perhaps not even *taught* (see Cohen & Ball, 1990a, 1990b). Teaching in these ways is hard. Thinking about teachers as the creators, not the implementors, of these *Standards*-inspired practices, alters the implementation issues in some fundamental ways. For example, this implies that *training* is an inappropriate metaphor. A teacher cannot be trained to know when to pursue an idea in more depth, when to let a student struggle, when to provide clarification. Training cannot equip teachers to develop a certain kind of classroom culture, or to consistently select worthwhile mathematical tasks. Neither is orchestrating a discussion in ways that are sensitive to gender or race, to other kinds of differences, *and* to some particular mathematical aims a matter of training.

Just as practitioners' needs are not for training, parents, school board members, and the wider society need something other than "updates" and information sessions or pamphlets. Because new ideas are necessarily filtered through listeners' existing frames of reference, their current knowledge and assumptions will shape what they understand of these ideas (Weiss & Cohen, 1991). Saying that children should learn to "solve mathematical problems" may conjure up images of solving "problems" such as 3568 x 1.002. That students should make conjectures, investigate, develop proofs, and argue will probably not illuminate the shifts in the nature of classroom discourse. And many will remember, without fondness, the debacle of the new math and worry that this is little more than a reincarnation of that last wave of curriculum reform.

If the *Standards* are to influence the directions of mathematics education, new standards are needed for the aims and means of implementation. Abandoning an instrumental view of how the standards might be translated into classroom work is crucial. The *Standards* have a contribution to make, but it is in thoughtful *sup*plementation to—not overturning of—practice (Weiss & Cohen, 1991). Supplementation implies the provision of new ideas, methods, and materials—all resources of practice. It also implies refinement and alteration of existing ideas, assumptions, and practices. And sometimes supplementation entails direct challenge to existing ways. Weiss and Cohen conclude that "old knowledge, by virtue of its extensive accumulation [and] confirmation by experience . . . necessarily dominates. . . . But people do learn, change their minds, see things in new ways" (p. 8). What can we do about supporting teachers in revising their ideas and practices?

Teachers and teacher educators will be the key agents of change and should be recognized and supported as such. They will need opportunities to learn new things—to develop their own understandings of mathematics in ways that enable them to listen to and

extend students' ideas, as well as to develop new sensitivities to each of their students and their ways of knowing and interacting. There will be subtle skills of observation and insight to develop in order to be able to orchestrate classroom discussions and group work in ways that are productive mathematically. Practitioners may need, in many contexts, to develop increased conviction and assertiveness in order to claim their right to do things differently. The uncertainty of practice itself, combined with teachers' sense that they do not have authority and power to work for change, means that they may have difficulty working experimentally and responsibly to develop their practice. They may also not know *how* to take a more experimental approach to their work, for the pressure to appear competent, smooth, and sure of one's methods and results predominates. Thoughtfully constructed curriculum materials, articles describing teaching and efforts by others to try particular ways of working, would further comprise useful resources for constructing new ways of working with students at all levels of mathematics education.

All in all, teachers will need a variety of opportunities to learn, and their work would be enhanced if there were more accessible ways to connect with teachers and teacher educators in other communities—to watch them teach, to talk with them about their work, to share ideas, questions, and frustrations. Can networks be established that make ongoing professional exchanges feasible, cheap, and not time-intensive? Can video footage from different kinds of math classes be developed and made available in ways that would be productive—and consistent with the idea of supplementing teachers' work and ways of thinking? Can multiple kinds of exemplars and data be made easily available—opening the proverbial classroom door to offer practitioners opportunities to learn and to build a sense of professional community?

A second—and critical—dimension of support is to communicate in educative ways with parents and other community members—indeed, with the wider public. Video materials could be developed which would provide images of the reform movement's ideas, and highlighting both the need for change and the uncertainty of accomplishing it. Whatever the mechanism, it is crucial to take seriously the need to talk with noneducators about these ideas, and about what it would take for teachers to manage to pull this off. While the classroom (and the cross-classroom community, if such a thing could be developed and supported) is the terrain for the actual work, if all support concentrates at that level of the challenge, the possibilities of change will be substantially diminished.

NCTM's accomplishments have been widely acclaimed. Other subject-matter organizations are now trying to emulate the lead of the mathematics education community. However, beyond the enthusiasm it has engendered, the initiative is still in its infancy. If

the *Standards* become something mechanical to be implemented, the initiative will probably fail. What these standards *do* remains to be seen. It lies ahead as they are interpreted and used in the ongoing efforts to reform practice.

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